2 Ways of Writing Sets

Roster Form	Set Builder Form
$A = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$	$B \rightarrow$ First 1000 natural numbers
	$B = \{x : x \in \text{Natural No.}, x \leq 1000\}$

- $F = \{x : x = 2n, n \in \text{Natural No.}, 2 \leq n \leq 7\}$
Write in Roaster Form.

$F = \{4, 6, 8, 10, 12, 14\}$

- $E = \{\frac{x}{2} : x \in \text{Positive Integers}, x < 5\}$

Write in Roaster Form.

$E = \{0.5, 1, 1.5, 2\}$

Types of Sets

1. Finite & Infinite sets

A set that contains countable number of element ~~so~~ is alled finite set.

&

Set of Uncountable number of elements is called infinite set.

2. Singleton Set

A set that contains only one element is called single ton set.

3. Null Set / Empty Set / Void Set

A ~~st~~ set that ~~contains~~ has no element is called as null set.

$\{\ \}$ or ϕ (Phi)

4. Equivalent & Equal Sets

$A = \{a, b, c\}$ $B = \{1, 2, 3\}$

$n(A) = n(B)$

$A \neq B$ [Equivalent Set]

$A = \{a, b, c\}$ $C = \{a, c, b\}$

$n(A) = n(B)$

$A = C$ [Equal set]

All equal sets are equivalent sets but not all equivalent sets are equal sets.

Subset & Superset (Beta aur Baap)

$A = \{1, 2, 3, 4, 5\}$ → Superset

$B = \{2, 4, 5\}$ → Subset

$A \supset B$ → A is a superset of B

$B \subset A$ → B is a subset of A

NUMBER

- Real
  - Rational
    - Integer
      - 1, 2, 3, 4, ... (positive Integer / Natural No.)
      - −1, −2, −3, −4, ... (Negative Integer)
      - 0 (0, 1, 2, ... n Whole No.)
    - Fractions
  - Irrational
- Imaginary

□ $x = \{1, 2, 3\}$ Write the subsets

No. of subsets $\rightarrow 2^3 = 8$

$\{\}, \{1\}, \{2\}, \{3\}, \{1,2\}, \{1,3\}, \{2,3\}, \{1,2,3\}$

Improper Subset

No. of Subsets $\rightarrow 2^n$

No. of Proper Substs $\rightarrow 2^n - 1$

No. of Non-Empty Proper Subsets $\rightarrow 2^n - 2$

~~Operation~~

Operators on Set

Union ($\cup$) $\rightarrow$ or, atleast one

Intersection ($\cap$) $\rightarrow$ Common, both, and

□ $A = \{1, 2, 4, 7, 8\}$

$B = \{1, 3, 5, 6, 8, 9\}$

$A \cup B = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$

$A \cap B = \{1, 8\}$

$A = \{1, 5, 6, 8, 10\}$

$B = \{1, 2, 4, 7, 10\}$

$C = \{2, 3, 8, 10, 11\}$

$B \cap C = \{2, 10\}$

$A \cup B = \{1, 2, 4, 5, 6, 7, 8, 10\}$

$(A \cap C) \cup B = \{8, 10\} \cup \{1, 2, 4, 7, 10\}$

$= \{1, 2, 4, 7, 8, 10\}$

$(A \cap B) \cap C = \{1, 10\} \cap C$

$= \{10\}$

$A = \{2, 5, 7, 8, 10\}$

$B = \{1, 2, 5, 6, 10\}$

$A \cup B = \{1, 2, 5, 6, 7, 8, 10\}$

$A \cap B = \{2, 5, 10\}$

$$\left.\begin{array}{ll} A \subset A \cup B, & B \subset A \cup B \\ A \cap B \subset A, & A \cap B \subset B \end{array}\right\}$$

$A \cap B \subset A \cup B$

- $A \cup A = A$
- $A \cap A = A$
- $A \subset B$
- $A \cup B = B$
- $A \subset B$

  $A \cap B = A$

Universal & Compliment Set (दादा जी)

$U = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$

$A = \{1, 2, 5, 6, 8, 10\}$

$B = \{1, 3, 5, 8\}$

★ $A' \,/\, \bar{A} \,/\, A^c \rightarrow$ A Compliment

$$A' = \{3, 4, 7, 9\}$$
$$B' = \{2, 4, 6, 7, 9, 10\}$$

★ $A \cup A' = U$ $\qquad$ $A \cap A' = \phi$

★ $(A')' = \{1, 2, 5, 6, 8, 10\}$

OR

$(A')' = A$

★ $A - B = \{2, 6, 10)$

$B - A = \{3\}$

FORMULAS

$A \cup A' = U$ $\qquad$ $A \cap A' = \phi$

1. $A - B = A \cap B' = A - A \cap B$
2. $B - A = A' \cap B = B - A \cap B$
3. $(A \cup B)' = A' \cap B'$
4. $(A \cap B)' = A' \cup B'$

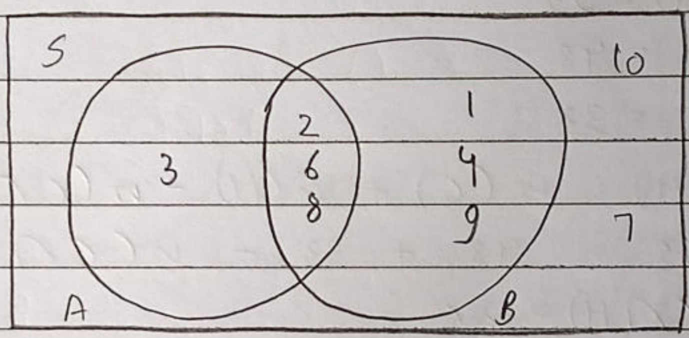
$\left.\begin{array}{l} 3 \\ 4 \end{array}\right\}$ De Morgan's Law

Venn Diagram

$U = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$

$A = \{2, 3, 6, 8\}$

$B = \{1, 2, 4, 6, 8, 9\}$



$A' = \{1, 4, 5, 7, 9, 10\}$

$B' = \{3, 5, 7, 10\}$

$A \cup B = \{1, 2, 3, 4, 6, 8, 9\}$

$(A \cup B)' = \{5, 7, 10\}$

$A \cap B = \{2, 6, 8\}$

$(A \cap B)' = \{1, 3, 4, 5, 7, 9, 10\}$

Only $A \leftarrow A - B = \{3\}$

Only $B \leftarrow B - A = \{1, 4, 9\}$

$$\boxed{n(A \cup B) = n(A) + n(B) - n(A \cap B)}$$

Ex. Total 70 students

48 cricket

33 Hockey

15 None of the games

$70 - 15 = 55$ Plays atleast one game. $\cup$

$n(C \cup H) = 55$

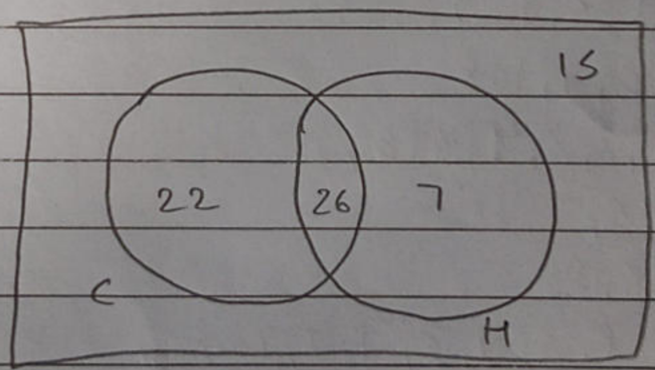
$n(C) = 48$

$n(H) = 33$

$n(C \cup H) = n(C) + n(H) - n(C \cap H)$

$55 = 48 + 33 - n(C \cap H$

$n(C \cap H) = 26$



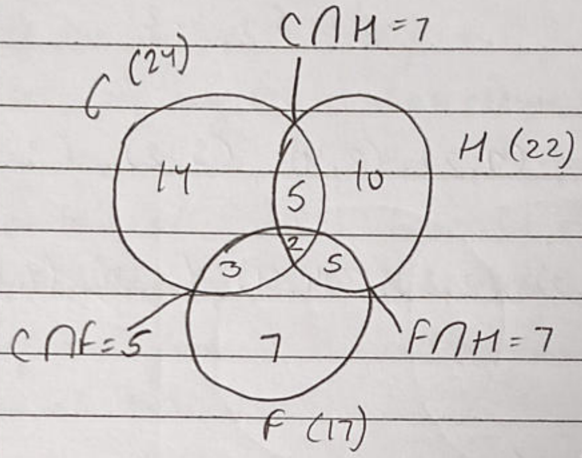
and $\cap$, or $\cup$

3 Sets Formula

$$n(A \cup B \cup C) = n(A) + n(B) + n(C) - n(A \cap B) - n(B \cap C) - n(C \cap A) + n(A \cap B \cap C)$$

Total $\rightarrow$ 50

Example-



$n(C) = 24$

$n(H) = 22$

$n(F) = 17$

$n(C \cap H) = 7$

$n(H \cap F) = 7$

$n(C \cap F) = 5$

4 who play none of the 3 games

$n(C \cup H \cup F) = 46$

$46 = 24 + 22 + 17 - 7 - 7 - 5 + n(C \cap H \cap F)$

$\boxed{n(C \cap H \cap F) = 2}$

Types of Questions:

How many students play

(i) Only 1 game $(14 + 10 + 7) = 31$

(ii) Atleast 2 games $(5 + 5 + 3 + 2) = 15$

(iii) Only 2 games $(5 + 5 + 3) = 13$

(iv) Cricket & Hockey but not Football $= 5$

(v) Atmost one game $(14 + 10 + 7 + 4) = 35$

Relation

Product Set → $n(A \times B) = n(A) \times n(B)$

$n(A \times B \times C) = n(A) \times n(B) \times n(C)$

$A = \{1, 2, 3\}$ $\quad$ $B = \{2, 4\}$

$A \times B = \{(1,2), (1,4), (2,2), (2,4), (3,2), (3,4)\}$

$B \times A = \{(2,1), (2,2), (2,3), (4,1), (4,2), (4,3)\}$

$n(A \times B) = 6$

$n(A) = 3$

$n(B) = 2$

$n(A \times B) = n(A) \times n(B)$

$n(A \times B) = n(B \times A)$

$A \times B \neq B \times A$

Equal & Equivalence defination

Types of Relations

Reflexive | Symmetric | Transitive

Reflexive	Symmetric	Transitive
If the product set or any of its subsets contain all possible pairs of the form (a, a), then it is called Reflexive (R)	If the product ~~set~~ has element (a, b) then it must have (b, a) also, then only it is called Symmetric	If the product has (a, b) & (b, c) then (a, c) must also be present then it is called as Transitive.

- 'is ∥ to' → Equivalence
- 'is ⊥ to' → Symmetric
- 'is greater than' or 'is less than' → Transitive
- 'is equal to' → Equivalence

Ex. {(1, 1) (1, 2) (1, 3) (2, 2) (3, 3)}

(a, a) it is Reflexive

Ex. {(1, 1) (2, 1) (1, 2)}

It is symmetric but not Reflexive

Ex. {(1, 1) (2, 2) (2, 1) (1, 2) (2, 2) (3, 3)}

It is both symmetric and Reflexive

Ex. {(1, 1) (2, 2) (1, 3) (3, 2) (1, 2)}

(1, 3) (3, 2) → (1, 2)

(1, 1) (1, 3) → (1, 3)

(3, 2) (2, 2) → (3, 2)

Ex. $\{(1,1)\ (1,2)\ (2,2)\ (2,1)\}$

It is Reflexive, transitive & symmetric relation

$R + S + T \rightarrow$ Equivalence

Q. The relation 'is equal to' over the set of all real number is

(A) R (B) S (C) T (D) E ✓

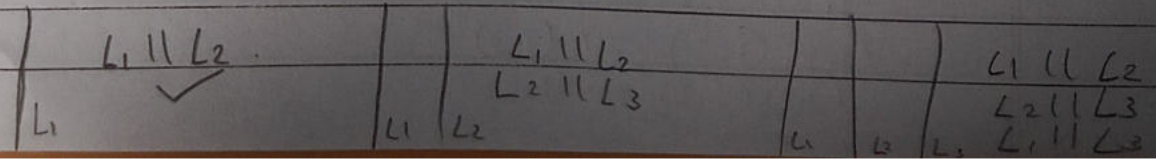
$R \rightarrow x = x$
$S \rightarrow x = y \Rightarrow y = x$
$T \rightarrow x = y,\ y = z \Rightarrow x = z$

Q. The relation 'is less than' over the set of all natural nos is

(A) R (B) S (C) T ✓ (D) E

$R \rightarrow 2 < 2$ ✗
$S \rightarrow 2 < 5 \Rightarrow 5 < 2$ ✗
$T \rightarrow 2 < 5,\ 5 < 8 \Rightarrow 2 < 8$ ✓

Q. The relation 'is parallel to' over the set of all straight lines is
(A) R (B) S (C) T (D) E ✓



Functions

$$Y = 2x + 5$$

$$f(x) = 2x + 5 \qquad y = f(x)$$

Y is dependent variable and x is independent variable

Examples:

- $f(x) = x^2 - 3x + 7$
  $f(4) = 4^2 - 3\times 4 + 7$
  $f(4) = 16 - 12 + 7$
  $f(4) = 11$

- $f(x) = x^2 - x + 3$
  $f(3/2) = (3/2)^2 - 3/2 + 3$
  $= 9/4 - 3/2 + 3$
  $= \frac{9 - 6 + 12}{4}$
  $= 15/4$

- $f(x) = x^2 - x + 7$
  $f(3x) = (3x^2) - 3x + 7$
  $= 9x^2 - 3x + 7$

- $f(x) = x^2 - 6x + 5$
  $f(x+3) = (x+3)^2 - 6(x+3) + 5$
  $= x^2 + 9 + 6x - 6x - 18 + 5$
  $= x^2 - 4$

- $f(x) = 3x + 5$
  and $f(2x) - 3(fx) = 0$ find $x = ?$

  $6x + 5 - 9x - 15 = 0$
  $-3x - 10 = 0$
  $3x + 10 = 0$
  $\boxed{x = \frac{-10}{3}}$

- Find the value of $f(2x) - 2f(x) + 8$ if $f(x) = 3x - 7$

  $\cancel{6x} - 7 - \cancel{6x} + 14 + 8$

  15

Concept of Domain & Range

The value of function x & y cannot be ∞ / $-\infty$.

The function cannot be $\sqrt{-ve}$ number.

$$y = \frac{1}{x}$$

$x \rightarrow$ Real No. $\{0\}$ Domain

The set of all possible values that x can take in any function is called as Domain of the function.

Type 1

□ $f(x) = \frac{1}{x+2}$ Domain $\rightarrow$ Real no. $- \{-2\}$

Range ↗ ↱ Domain

□ $y = x^2 + 1$

Domain $\rightarrow$ All real no.

Range $\rightarrow$ All Real No. ≥ 1

x	y
0	1
2	5
-3	10
-0.5	1.25

Type 2 If a product has following relation

$\{(1,2)\ (3,5)\ (4,6)\}$

Domain $\rightarrow \{1, 3, 4\}$

Range $\rightarrow \{2, 5, 6\}$

Type 3 If $f(x) = 2^{x+1}$ and the value of x lies between $1 \leq x \leq 6$. Find the range of value of $f(x)$.

$$f(1) = 2^{1+1} = 2^2 = 4$$
$$f(6) = 2^{6+1} = 2^7 = 128$$

$4 \leq y \leq 128$ or $4 \leq f(x) \leq 128$

Ex. $f(x) = \log_{10}(1+x)$

$0 \leq x \leq 999$

$$\log_{10}(1) = 0$$
$$\log_{10}(1000) = 3$$

$$0 \leq f(x) \leq 3$$

Types of Functions

1. Composite function ~~(f. of (fof, gof~~ (fof, fog, gof etc.) :-

$$f(x) \to fof \to f(f(x))$$

Ex. $f(x) = 2x + 7$

$$fof = f(f(x)) = 2f(x) + 7 = 2(2x+7) + 7$$
$$= 4x + 14 + 7$$
$$= 4x + 21$$

Ex. $f(x) = 5x - 8$, then find fof

$$f(f(x)) = 5f(x) - 8$$
$$= 5(5x-8) - 8$$
$$= 25x - 40 - 8$$
$$= 25x - 48 \qquad = 27$$

Ex

Ex. $f(x) = 2x-5$ $g(x) = 4x+1$

Find $f \circ g$, $g \circ f$?

$f \circ g \to f(g(x)) = 2g(x) - 5$

$= 2(4x+1) - 5$

$= 8x + 2 - 5$

$= 8x - 3$

$g \circ f \to g(f(x)) = 4f(x) + 1$

$= 4(2x-5) + 1$

$= 8x - 20 + 1$

$= 8x - 19$

Short Cut

2. Inverse function $(f^{-1}(x))$

Ex $f(x) = \dfrac{x+5}{2} \rightarrow y = \dfrac{x+5}{2}$

$x = \dfrac{y+5}{2}$

$y = 2x - 5 \quad (fx^{-1}(x))$

$\boxed{\text{x ki jagah y \& y ki jagah x then solve for x}}$

Ex. $f(x) = 4x+5$, then find $f^{-1}(x)$.

$y = 4x+5$

$x = 4y+5$

$y = \dfrac{x-5}{4}$

Ex. $f(x) = \dfrac{2x-1}{3x}$, $f^{-1}(x)$.

$x = \dfrac{2y-1}{3y}$

$3xy = 2y - 1$

$1 = 2y - 3xy$

$1 = y(2-3x)$

$\dfrac{1}{2-3x} = y$

$f(x) = \dfrac{x+3}{3x+5}$

$x = \dfrac{y+3}{3y+5}$

$3xy + 5x = y + 3$

$5x - 3 = y - 3xy$

$5x - 3 = y(1-3x)$

$\dfrac{5x-3}{1-3x} = y$

3. One one / Onto / Into Function

One One → If for every value of x there is a unique value of y, then it is called as one-one function / one-one mapping.

$$f(x) = 4x$$

x	1	2	-5	$\frac{1}{2}$
y	4	8	-20	2

Onto → If for every value of y there is atleast one value of x, then it is onto function. Onto function is also called as surjective function. A function which is both one-one & onto is called bijective function.

$$f(x) = x^2$$

x	1	-1	2	-2
y	1		4	

Into → If there is atleast one value of y for which there is no value of x, then it is into function.

$$f(x) = \frac{1}{x}$$

x	$\frac{1}{5}$	5	0.1	
y	5	0.2	10	0 – nd ∞

4. Constant Function

$f(x) = k$ is called constant function

$y = 5$

$\therefore$ $y = k$ is a function but

$x = k$ is not a function

$\{(x, p)\ (y, p)\ (z, p)\}$ this is a function

$\{(x, p)\ (x, q)\ (x, r)\}$ this is not a function

x ki do alag alag value ke liye y ki single value ho skti hai

lekin

y ki do alag alag value ke liye x ki single value nhi ho skti